Available online at www.sciencedirect.com

SciVerse ScienceDirect

Procedia Engineering 15 (2011) 1229 – 1233

Procedia Engineering

www.elsevier.com/locate/procedia

Advanced in Control Engineeringand Information Science

Robust adaptive control for a class of chaotic system using backstepping

Naibao He [a,b] , Qian Gao[a] ,Changsheng Jiang [b]a*

[a]*Huaihai Institute of Techology, Lianyungang 222005, China*
[b]*College of Automatic Engineering,Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China*

Abstract

In this paper, an approach for chaotic system is proposed using adaptive backstepping with tuning functions. Firstly, based on coordinate transform, the paper deduced the principle with which Chua's chaotic system can be translated into the so-called general strict-feedback form. Secondly, an adaptive parameter control law combining the backstepping method with robust control technology is developed and thus the output tracking is successfully accomplished for the system with unknown parameters and dynamic uncertainties. It is proved that the derived robust adaptive controller based on Lyapunov stability theory can guarantee that all states of the closed-loop system are globally uniformly ultimately bounded, and lead the system tracking error to a small neighborhood. Finally simulation results are provided to show the effectiveness of the proposed approach.

Selection and/or peer-review under responsibility of [CEIS2011]

*Key Words:*Chua's chaotic system; backstepping; adaptive control ;output-tracking

1. Introduction

In recent adaptive and robust control literatures for nonlinear systems, backstepping has become one of the most popular design methods for adaptive nonlinear control because it can guarantee global stabilities, tracking, and transient performannce for a broad class of strict-feedback system. In this paper, we firstly show that the Chua's system in the research of chaos can be transformed into a kind of nonlinear system in the so-called general strict feedback form. Secondly, we focus on realizing chaos synchronization of a class of uncertain system by using only one controller and estimate parameters

* Corresponding author. Tel.: 13815655599;.
E-mail address: henaibao@126.com.

doi:10.1016/j.proeng.2011.08.227

update law. The method adopted here is adaptive backstepping design procedure. Finally, numerical simulations are provided to show the effectiveness of the proposed approach

2. Problem statement

Consider the follow the nonlinear circuit

$$\begin{cases} \dot{y}_1 = p_1(y_2 - y_1 - p_4 y_1 + \frac{1}{2}(p_3 - p_4)(|y_1+1| - |y_1-1|)) \\ \dot{y}_2 = y_1 - y_2 + y_3 \\ \dot{y}_3 = -p_2 y_2 \end{cases} \tag{1}$$

It is clear that they can be transformed into the desired feedback form as follows.

Let $x_1 = y_3$ $x_2 = y_2$ $x_3 = y_1$ then (1) can be rewritten as

$$\begin{cases} \dot{x}_1 = -p_2 x_2 \\ \dot{x}_2 = x_1 - x_2 + x_3 \\ \dot{x}_3 = u + p_1 x_2 - p_1(1+p_4)x_3 - \frac{1}{2} p_1(p_3 - p_4)(|x_3+1| - |x_3-1|) \end{cases} \tag{2}$$

Let $g_1 = -p_2, g_2 = 1, g_3 = 1$, $\theta = 0, \Delta_1 = 0$, $\Delta_2 = x_1 - x_2$

$$\Delta_3 = p_1 x_2 - p_1(1 + p_4 x_3) - \frac{1}{2} p_1(p_3 - p_4)(|x_3+1| - |x_3-1|)$$

In this way, the controller Chua's circuit (1) can be easily written into a 3-order feedback form as follows.

$$\begin{cases} \dot{x}_i = g_i(\overline{x}_i, t)x_{i+1} + \theta^T \phi_i(\overline{x}_i, t) + \Delta_i(\overline{x}_i, t) \\ \qquad 1 \le i \le 2 \\ \dot{x}_3 = g_3(\overline{x}_3, t)u + \theta^T \phi_3(\overline{x}_3, t) + \Delta_3(\overline{x}_3, t) \\ y = x_1 \end{cases} \tag{3}$$

where, $x = [x_1, x_2, x_3]^T \in R^3$, $u \in R$, and $y \in R$ are the states, input and output, respectively; $\overline{x}_i$ denotes $\overline{x}_i = [x_1, \cdots, x_i]^T \in R^i$, $i = 1,2,3$; $\theta = [\theta_1, \theta_2, \cdots, \theta_l]^T \in R^l$ is the vector of unknown constant parameters of interest; $\Delta_i(\overline{x}_i, t)$ is unknown Lipschitz continuous dynamic uncertain functions; $g_i(\cdot) \ne 0$, $\phi_i(\cdot)$, $i = 1,2,3$ are known smooth nonlinear functions,with their j th derivatives uniformly bounded in t.

In this paper, the problem is to design an adaptive algorithm and an adaptive state-feedback controller for the system (3) to asymptotically track the output $y_r = x_r(t)$ of the reference model, i.e.

$$|y(t) - y_r(t)| \le \varepsilon, \quad as \quad t \to \infty$$

3. Adaptive controller design

Throughout the paper, the following assumptions are made on system (1).

Assumption1: For each $1 \le i \le n$, there exists an unknown positive constant $\overline{p}_i$ such that, for all $\Delta_i(\overline{x}_i, t)$, satisfies $\left|\Delta_i(\overline{x}_i, t)\right| \le \overline{p}_i \psi_i(x_1, \cdots, x_i)$, where $\psi_i(x_1, \cdots, x_i)$ is a known nonnegative smooth function.

Assumption2: Let y_r be a bounded reference signal whose n-th derivatives is also bounded, and for random known positive constant d, satisfies $\left|y_r^{(n)}(t)\right| \le d$

Lemma 1[8]: For any x and y in R^n, and for any positive real number δ, we have

$$x^T y \le \frac{1}{4\delta}|x|^2 + \delta|y|^2$$

In order to design an adaptive algorithm to achieve the objective, adaptive backstepping with tuning functions is employed. The backstepping design procedure contains n steps. At step i, an intermediate control function α_i shall be developed by using an appropriate Lyapunov function V_i. Let us first consider the equation when $i = 1$.

Step1. Define the first error variable

$$z_1 = x_1 - y_r \tag{4}$$

Its derivative is given by $\dot{z}_1 = \dot{x}_1 - \dot{y}_r$

We consider the Lyapunov function candidate

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}\tilde{\theta}_1^T \Gamma^{-1} \tilde{\theta}_1 + \frac{1}{2} r^{-1} \tilde{p}_1^2 \tag{5}$$

The time derivative of the Lyapnuov function V_1 satisfies

$$\begin{aligned}\dot{V}_1 &= z_1 \dot{z}_1 - \tilde{\theta}_1^T \Gamma^{-1} \dot{\hat{\theta}}_1 - r^{-1} \tilde{p}_1 \dot{\hat{p}}_1 \\ &= z_1(\dot{x}_1 - \dot{y}_r) - \tilde{\theta}_1^T \Gamma^{-1} \dot{\hat{\theta}}_1 - r^{-1} \tilde{p}_1 \dot{\hat{p}}_1 \\ &= z_1(g_1 x_2 + \theta_1^T \phi_1(x_1) + \Delta_1 - \dot{y}_r) - \tilde{\theta}_1^T \Gamma^{-1} \dot{\hat{\theta}}_1 - r^{-1} \tilde{p}_1 \dot{\hat{p}}_1\end{aligned}$$

Where $\Gamma = \Gamma^T > 0$ is the adaptive gain matrix , $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, $\tilde{p}_i = p_i - \hat{p}_i$, $\hat{\theta}_i$ and $\hat{p}_i$ represent the estimate vector of unknown parameter vector θ_i and p_i, respectively ,r is a design parameter.

According to assumption 1, assumption 2 and lemma 1, we have

$$\begin{aligned}\dot{V}_1 \le\ & z_1(g_1 x_2 + \hat{\theta}_1^T \phi_1(x_1)) + 2\delta + \frac{z_1}{4\delta}(d^2 + \hat{p}_1 \psi_1^2(x_1)) \\ & - \Gamma^{-1} \tilde{\theta}_1^T (\dot{\hat{\theta}}_1 - \Gamma z_1 \phi_1(x_1)) - r^{-1} \tilde{p}_1 (\dot{\hat{p}}_1 - r \frac{z_1^2}{4\delta} \psi_1^2(x_1))\end{aligned} \tag{6}$$

Let $z_2 = x_2 - \alpha_1$, and α_1 is a virtual control law to be defined as follows:

$$\alpha_1 = \frac{1}{g_1}(-c_1 z_1 - \hat{\theta}_1^T \phi_1(x_1)) - \frac{z_1}{4\delta}(d^2 + \hat{p}_1 \psi_1^2(x_1)) \tag{7}$$

Substituting (7) into(6), we obtain

$$\begin{aligned}\dot{V}_1 \le\ & -c_1 z_1^2 + z_1 g_1 (x_2 - \alpha_1) + 2\delta \\ & - \Gamma^{-1} \tilde{\theta}_1^T (\dot{\hat{\theta}}_1 - \Gamma z_1 \phi_1(x_1)) - r^{-1} \tilde{p}_1 (\dot{\hat{p}}_1 - r \frac{z_1^2}{4\delta} \psi_1^2(x_1))\end{aligned}$$

To cancel the last two terms in the above derivative, we choose the update law

$$\dot{\hat{\theta}}_1 = \Gamma z_1 \phi_1(x_1) \ ,$$

$$\dot{\hat{p}}_1 = r\frac{z_1^{\ 2}}{4\delta}\psi_1^{\ 2}(x_1) \tag{8}$$

which yields

$$\dot{V}_1 \le -c_1 z_1^{\ 2} + g_1 z_1 z_2 + 2\delta \tag{9}$$

Step n. similar to the last step ,Define Lyapunov function candidate as

$$V_n = V_{n-1} + \frac{1}{2}z_n^{\ 2} + \frac{1}{2}\tilde{\theta}_n^{\ T}\Gamma^{-1}\tilde{\theta}_n + \frac{1}{2}r^{-1}\tilde{p}_n^{\ 2} \tag{10}$$

Its derivative is

$$\dot{V}_n = \dot{V}_{n-1} + z_n\dot{z}_n - \tilde{\theta}_n^{\ T}\Gamma^{-1}\dot{\hat{\theta}}_n - r^{-1}\tilde{p}_n\dot{\hat{p}}_n$$

The choice of control u is given by

$$\begin{aligned} u = \alpha_n = \frac{1}{g_n}\Big(&-c_n z_n + \sum_{j=1}^{n-1}\frac{\partial\alpha_{i-1}}{\partial x_j}g_j x_{j+1} + \sum_{j=1}^{n-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\theta_j^T\phi_j(x_1,\cdots;x_j)) - \hat{\theta}_n^T\phi_n(x_1,\cdots;x_n) \\ &-\frac{z_n}{4\delta}\hat{p}_n\psi_n^2(x_1,\cdots;x_n) - \frac{z_n}{4\delta}d^2(\frac{\partial\alpha_{n-1}}{\partial y_r}) + \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_n}\dot{\hat{\theta}}_n + \frac{\partial\alpha_{n-1}}{\partial\hat{p}_n}\dot{\hat{p}}_n\Big) \end{aligned} \tag{11}$$

Selecting the update laws for $\dot{\hat{\theta}}_n$ and $\dot{\hat{p}}_n$,

$$\dot{\hat{\theta}}_n = \Gamma z_n\phi_n(x_1,\cdots,x_n)\,,\ \dot{\hat{p}}_n = \frac{z_n^{\ 2}}{4\delta}r\psi_n^{\ 2}(x_1,\cdots,x_n) \tag{12}$$

then

$$\dot{V}_n \le -\sum_{k=1}^{n}c_k z_k^{\ 2} + 2n\delta \tag{13}$$

The c_n selected is the smallest positive candidate constant, μ_n is a positive candidate bounding number, then we have

$$\dot{V}_n \le -c_n V_n + \mu_n \tag{14}$$

let $\lambda = \mu_n / c_n$, then $0 \le V_n(t) \le \lambda + [V_n(0) - \lambda]e^{-c_n t}$

By means of the Barbalat lemma, we have

$$\lim_{t\to\infty} V_n = 0 \tag{15}$$

So far the entire design procedure is completed.

Theorem1: Under assumption 1, assumption 2 and lemma 1, there exists a state feedback controller u and adaptive control law $\dot{\hat{\theta}}_i = \Gamma z_i\phi_i(x_1,\cdots,x_i)$ $\dot{\hat{p}}_i = \frac{z_i^{\ 2}}{4\delta}r\psi_i^{\ 2}(x_1,\cdots,x_i)$ acting on system. The closed-loop system is globally stable for all admissible uncertainties, and its output tracking error satisfies

$$|e| = |y - y_r| \le \sqrt{2V_n(0)}e^{-\frac{c_n t}{2}} + \sqrt{2\mu_n / c_n}$$

4. Conclusion

We have presented an adaptive control design procedure for the regulation of nonlinear systems with both parametric uncertainty and unknown nonlinearities. In the design, backstepping method is used to get the parameter adaptive control law. The proposed systematic backstepping design method has been proved to be able to guarantee global uniform ultimate boundedness of closed-loop signals. In addition, the output of the system has been proven to converge to an arbitrarily small neighborhood of the origin. Simulation results have been provided to show the effectiveness of the proposed approach

References

[1] Yu YG, Zhang SC. Adaptive backstepping control of the uncertain Lū system,.Chin Phys,2002,l.11(12), 1249-1253.

[2] Y.-S. Zhong, Globally stable adaptive system design for minimum phase SISO plants with input saturation, Automatica, 2005,41(9). 1539-1547.

[3] C.L. Yang, R.Y. Ruan and M.K. Gong, Robust adaptive state feedback controller design and analysis of a class of uncertain nonlinear systems,Acta Mathematica Scientia, 2004.24(1).26-37.

[4] Jiang Z P,Parly L, Design of robust adaptive controller for nonlinear systems with uncertainties, Automatica, 1998. 34(7), 825- 840.

[5] Krishnamurthy P, Khorrami F, Jiang Z P,Global output feedback tracking for nonlinear systems in generalized out-feedback canonical, IEEE Trans. On Automatic Control, 2002,.47(5) ,814-819.

[6] Hai-Bo Ji and Hong-Sheng Xi, Adaptive output-feedback tracking of stochastic nonlinear systems, IEEE Trans. On Automatic Control, 2006,.51(2),355-360.

[7] J. Zhou, C. Wen, Y. Zhang, Adaptive output control of a class of timevarying uncertain nonlinear systems, Nonlinear Dynamics System Theory, 2005,.5, 285-298.

[8] Krishnamurthy P, Khorrami F, Jiang Z P,Global output feedback tracking for nonlinear systems in generalized out-feedback canonical ,IEEE Trans. On Automatic Control, 2002,47(5), 814-819.